Complexity Theory (Winter 2017/18)	Problem Set #2 (Due 04.12.2017)

Problem 1.1. Provide a proof that there is a universal non-deterministic TM which, on input a non-deterministic machine M, input x, and a time-constructible function $T(n)$, can simulate $T(|x|)$ steps of M in time $O(T(|x|))$.

Problem 1.2. Prove that if $P = NP$ then $NP = coNP$.

Problem 1.3. Explain why the following argument does not show $P \neq NP$. Assume $P = NP$ and obtain a contradiction. If $P = NP$, then $SAT \in P$, and so for some k, $SAT \in DTIME(n^k)$. Since every language in NP is polynomial time reducible to SAT, we have $NP \subseteq DTIME(n^k)$. So $P \subseteq DTIME(n^k)$. But by the time hierarchy theorem, $DTIME(n^{k+1})$ contains a language that is not in $DTIME(n^k)$. This language is in P. This is a contradiction, so $P \neq NP$.

Problem 1.4. Prove the space hierarchy theorem. Where do you use space constructivity?

Problem 1.5. (1) Show that $SAT \in SPACE(n)$. (2) Show that $NP \neq SPACE(n)$. Why doesn't (2) contradict (1)? We do not know if either class is contained in the other! [Hint: Use a closure property of NP.]

Problem 1.6. Show that the oracle B constructed in the Baker-Gill-Solovay construction separating P^B from NP^B belongs to EXP.