Advanced Automata Theory (Summer 2017) Problem Set #5 (Due 28.06.2017)

Problem 1. Prove that an ω-language is co-Büchi recognizable if and only if it is recognizable by a deterministic co-Büchi automaton.

Hint: Try the following construction. Assume the co-Büchi automaton is $A = (Q, \Sigma, q_0, \delta, F)$. Construct a deterministic co-Büchi automaton $D = (2^Q \times 2^Q, \Sigma, (\{q_0\}, \{q_0\}), \delta', 2^Q \times \{\emptyset\})$ where

$$\delta((Q,S),a) = \begin{cases} \begin{cases} \{(T,T) \mid T = \delta(Q,a)\} & S = \emptyset \\ \{(T,T') \mid T = \delta(Q,a), T' = \delta(S,a) \setminus F\} & S \neq \emptyset \end{cases} \end{cases}$$

Problem 2. (a) Suppose $\mathcal{R}$ is a Rabin automaton with accepting pairs

$$\{(E_1, F_1), (E_2, F_2), \ldots, (E_n, F_n)\}$$

such that

$$E_1 \subseteq E_2 \subseteq \ldots \subseteq E_n.$$

Show that there is an equivalent set of accepting pairs

$$\{(E'_1, F'_1), (E'_2, F'_2), \ldots, (E'_n, F'_n)\}$$

such that, further,

$$E'_1 \subseteq F'_1 \subseteq E'_2 \subseteq F'_2 \subseteq \ldots \subseteq E'_n \subseteq F'_n$$

Such an accepting condition is called a *Rabin chain condition.*

Note that any Rabin automaton can be transformed into a Rabin chain automaton. However, if the requirement $E_1 \subseteq E_2 \subseteq \ldots \subseteq E_n$ does not hold, this construction also changes the transition structure of the automaton.

(b) Let $\mathcal{A}$ be an automaton and let $\lambda : Q \to \{0, 1, \ldots, k\}$ be a map from states to a set of *colors.*

The automaton $\mathcal{A}$ and the coloring λ accepts a word $w \in \Sigma^\omega$ by the *parity* accepting condition iff there is a run ρ such that the maximum color appearing infinitely often along the run is odd:

$$\max\{\lambda(q) \mid q \in \mathit{Inf}(\rho)\} \text{ is odd}$$

Show using part (a) that parity automata accept exactly the ω-regular languages.

(c) By changing the coloring, show directly that deterministic parity automata are closed under complementation.

Problem 3. Show directly that deterministic Muller automata are closed under union and intersection by giving a product construction.

Problem 4. Finish the proof sketched in class that the marked subset construction is sound, that is, if the marked subset automaton accepts a word then the original automaton also accepts the word.

Problem 5. Let A be a Streett automaton with n states and m accepting pairs. Give a nondeterministic Büchi automaton with $O(n2^m)$ states for $L(A)$.

Problem 6. Let A be a deterministic Büchi automaton. Let A' be the automaton obtained by applying a (finite-word automaton) minimization procedure on A. (a) What can we say about the ω-language accepted by A'? (b) Does this provide a minimal deterministic Büchi automaton?

Problem 7. A universal automaton accepts a word iff *all* runs of the automaton on the word are accepting. Show that universal co-Büchi automata accept exactly the ω-regular languages.