Advanced Automata Theory (Summer 2017) Problem Set #4 (Due 14.06.2017)

Problem 1. Let $U, V \subseteq \Sigma^*$ be languages. Prove or disprove (by giving a counterexample) the following propositions:

(a) $(U \cup V)^\omega = U^\omega \cup V^\omega$

(b) $(U^*)^\omega = U^\omega$

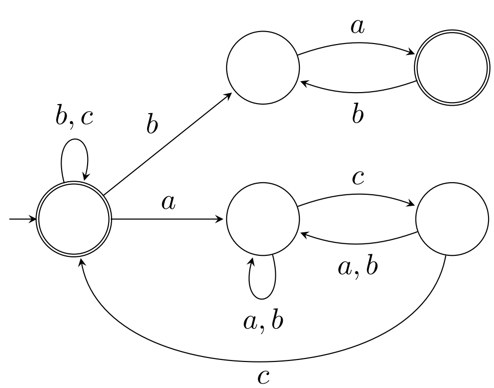
(c) $\lim U \subseteq U^\omega$ and $\lim U \supseteq U^\omega$

Now consider a language $U \subseteq \Sigma^*$ such that $U = U^*$. Again, prove or disprove the following propositions:

(d) $\lim U \subseteq U^\omega$ and $\lim U \supseteq U^\omega$

Problem 2. Let $\Sigma = \{a, b, c\}$.

(a) Consider the following Büchi-automaton $\mathcal{A}$. Give a representation of the language $L_1 = L(\mathcal{A})$ in form of ω-regular expressions (i.e., find expressions r_i, s_i over finite words such that L_1 is defined by $r_1 s_1^\omega + \ldots + r_n s_n^\omega$).



(b) Consider the ω-regular language $L_2 = U \cdot V^\omega$ with $U = (bb)^*$ and $V = \Sigma^* ba$.

Construct a Büchi-automaton which recognizes the language L_2. Is L_2 representable by a deterministic Büchi-automaton? If there is a deterministic Büchi-automaton, construct a regular language $W \subseteq \Sigma^*$ such that $L_2 = \lim W$. Otherwise, prove that it cannot be recognized by a deterministic Büchi-automaton.

Problem 3. Provide a construction that takes a deterministic Büchi automaton $\mathcal{A} = (Q, \Sigma, q_0, \Delta, F)$ and produces a nondeterministic Büchi automaton $\mathcal{B}$ accepting $\mathcal{A}$'s complement (i.e., $L(\mathcal{B}) = \Sigma^\omega \setminus L(\mathcal{A})$). Make sure that your construction results in a Büchi automaton of polynomial size.

Problem 4. A *generalized* Büchi automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, \mathcal{F})$ where Q, Σ, Δ, and q_0 are as in ordinary Büchi automata, and $\mathcal{F} = \{F_1, \dots, F_n\}$ is a collection of sets $F_i \subseteq Q$ of final states.

A run $\rho = q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2 \xrightarrow{a_2} \dots$ of a generalized Büchi automaton $\mathcal{A}$ on a word $\alpha = a_0 a_1 \dots \in \Sigma^\omega$ is accepting if $\mathit{Inf}(\rho) \cap F_i \neq \emptyset$ for each $i \in \{1, \dots, n\}$. The language $L(\mathcal{A})$ of a generalized Büchi automaton $\mathcal{A}$ is the set of all words $\alpha \in \Sigma^\omega$ for which an existing run exists.

Show that generalized Büchi automata recognize exactly the class of ω-regular languages. More precisely, given a generalized Büchi automaton $\mathcal{A}$, construct a Büchi automaton $\mathcal{B}$ with $L(\mathcal{B}) = L(\mathcal{A})$.

Problem 5. Show that there exists an alphabet Σ and a word $\alpha \in \Sigma^\omega$ such that $\{\alpha\}$ is not ω-regular.

Problem 6. Show that Buchi automata are closed under projection. A projection is a mapping $\pi \colon \Sigma \to \Gamma$, which maps symbols from an automaton's input alphabet Σ to a new alphabet Γ. A projection can be extended to words in the usual way: for $\alpha \in \Sigma^\omega$, we define $\pi(\alpha) = \pi(\alpha(0))\pi(\alpha(1))\pi(\alpha(2)) \ldots \in \Gamma^\omega$. The task is to show that for an ω-regular language $L \subseteq \Sigma^\omega$ and a projection $\pi \colon \Sigma \to \Gamma$, the language $\pi(L) = \{\pi(\alpha) \mid \alpha \in L\}$ is again ω-regular.

Problem 7. Prove or disprove: The set of languages recognized by deterministic Büchi automata is closed under union.

Problem 8. Let $\Sigma = \{a, b, c\}$. Construct Muller automata for the languages informally given below:

(a) "Whenever a occurs there is a b and a c at later positions."

(b) "If a occurs infinitely often, then every a is directly followed by a b."

Problem 9 (Optional).

(a) Show that there is a regular ω-language $L \subseteq \{a, b\}^\omega$ which cannot be recognized by a deterministic Muller automaton $\mathcal{A} = (Q, \{a, b\}, \delta, q_0, \mathcal{F})$ with $|\mathcal{F}| = 1$ (i.e., $\mathcal{F} = \{F\}$ for some $F \subseteq Q$). Give a complete proof.

(b) Give regular candidate ω-languages L_n that cannot be recognized by deterministic Muller automata with acceptance component $\mathcal{F}$ such that $|\mathcal{F}| \leq n$. A proof is not required.