Advanced Automata Theory (Summer 2017) Problem Set #2 (Due 17.05.2017)

Problem 1. Let ϕ_1 and ϕ_2 be MSO formulas. The following are fundamental decision problems for any logic:

1. *Satisfiability* ϕ_1 is satisfiable if there exists some word model that satisfies the formula.
2. *Validity* ϕ_1 is valid if every word model satisfies ϕ_1.
3. *Equivalence* ϕ_1 and ϕ_2 are equivalent if they satisfy the same set of word models.

Show that these problems are all decidable.

Problem 2. Show that every MSO formula is equivalent to one in which all second order variables are existentially quantified in the prenex normal form. [Hint: From the original formula, create the automaton and then convert the automaton back to MSO.]

Problem 3. Show how you can express the following constraints in MSO: $(X, Y, \dots$ stand for sets of positions)

1. X is not empty
2. X is a multiple of three
3. $Z = X \cup Y$, $Z = X \cap Y$
4. X is contained in Y and Y has one more element than X

If you used first order variables, can you also write formulas without any first order variables for the same formulas?

Problem 4. Is the formula

$$\forall x. \exists y. x < y \wedge Q_a(y)$$

satisfiable?

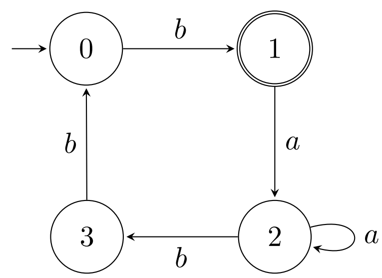
Suppose we define *infinite* words as functions $w : \mathbb{N} \to \Sigma$, where $\mathbb{N}$ is the set of natural numbers. An infinite word model is defined in the natural way. Can you give an infinite word model which satisfies the formula?

Problem 5. Consider the MSO formula

$$\exists X.\exists Y.\mathsf{NotEmpty}(X) \wedge \mathsf{NotEmpty}(Y) \wedge X \subseteq Q_a \wedge Y \subseteq Q_b$$

we discussed in the class. Here, $\mathsf{NotEmpty}(X)$ states that X is not empty. Construct an automata whose language is the same as the models of this formula. Instead of encoding $\mathsf{NonEmpty}$ using the "basic" predicates, you should directly construct an automaton for that predicate. (I hope the automaton does not blow up too badly!)

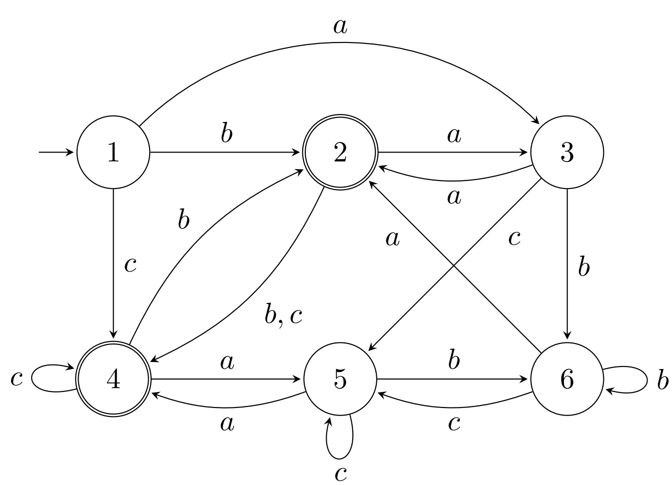
Problem 6. Let $\mathcal{A}$ be the following NFA:



We have seen in the lecture how to construct an equivalent MSO formula $\varphi_{\mathcal{A}}$ with *four* set variables X_0, X_1, X_2, X_3. Construct an equivalent MSO formula $\varphi'_{\mathcal{A}}$ with only *two* set variables. Can this idea be generalized to use $\log n$ variables?

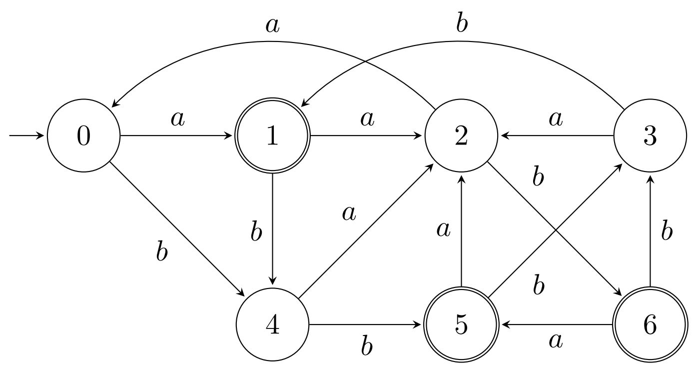
Hint: Find a suitable encoding that can encode the four different states of the NFA using only two set variables.

Problem 7. Consider the following DFA $\mathcal{A}$:



Provide (at least) one quotient automaton of $\mathcal{A}$ that is again a DFA. Also, give an equivalent regular expression for this DFA.

Problem 1.8. Apply the partition refinement algorithm presented in the lecture to minimize the following DFA $\mathcal{A}$:



Note down all intermediate computations in form of a tree as shown in the lecture. Also, draw the resulting DFA.