Parametricity and Modular Reasoning
Homework #5

Instructor: Derek Dreyer

Assigned:	Tuesday, 11 December 2012
Problem 2:	Thursday, 13 December 2012
Due:	Tuesday, 18 December 2012

1 Working with callcc

Suppose $\vdash e_0 : \mathrm{bool}$ and $k : \neg\tau \vdash e_1 : \tau$ and $k : \neg\tau \vdash e_2 : \tau$. Use the logical relation and parametricity to prove

$$\vdash \mathrm{callcc}(k.\mathrm{if}\ e_0\ \mathrm{then}\ e_1\ \mathrm{else}\ e_2) \equiv (\mathrm{if}\ e_0\ \mathrm{then}\ \mathrm{callcc}(k.e_1)\ \mathrm{else}\ \mathrm{callcc}(k.e_2)) : \tau.$$

2 Bind lemmas for Ahmed's step-indexed model

1. Prove bind for open terms. Specifically, assume $(n, e_1, e_2) \in E[\sigma]\rho$ and

$$\forall j \leq n. \forall v_1, v_2. (j, v_1, v_2) \in V[\sigma]\rho \implies (j, K_1[v_1], K_2[v_2]) \in E[\tau]\rho'$$

   and show $(n, K_1[e_1], K_2[e_2]) \in E[\tau]\rho'$.

2. Prove bind for open terms:

$$\frac{\Delta; \Gamma \vdash e_1 \precsim e_2 : \sigma \quad \Delta; \Gamma, x : \sigma \vdash K_1[x] \precsim K_2[x] : \tau \quad x \notin \mathrm{fv}(K_1, K_2)}{\Delta; \Gamma \vdash K_1[e_1] \precsim K_2[e_2] : \tau}$$

   Hint: You'll be given some n and the usual substitutions. Reduce the problem to

$$\begin{aligned} \forall j \leq n. \forall v_1, v_2.\ (j, v_1, v_2) \in V[\sigma]\rho \implies \\ (j, \delta_1\gamma_1 K_1[v_1], \delta_2\gamma_2 K_2[v_2]) \in E[\tau]\rho' \end{aligned}$$

   by applying the bind lemma for closed terms. You will need to use downard-closure to extend the context.