Parametricity and Modular Reasoning
Homework #4

Instructor: Derek Dreyer

Assigned: Tuesday, 27 November 2012
Due: Tuesday, 4 December 2012

Coincidence Lemmas

Recall the definition of the value relation at product and sum types:

$$V[\sigma \times \tau]\rho \stackrel{\text{def}}{=} \{(\langle v_1,\, v_1'\rangle, \langle v_2,\, v_2'\rangle) \mid (v_1, v_2) \in V[\sigma]\rho \wedge (v_1', v_2') \in V[\tau]\rho\}$$

$$V[\sigma + \tau]\rho \stackrel{\text{def}}{=} \begin{array}{l}\{(\text{inj}_1\, v_1,\, \text{inj}_1\, v_2) \mid (v_1, v_2) \in V[\sigma]\rho\} \cup \\ \{(\text{inj}_2\, v_1,\, \text{inj}_2\, v_2) \mid (v_1, v_2) \in V[\tau]\rho\}\,.\end{array}$$

Suppose

i. $\forall v_1, v_2.(v_1, v_2) \in E[\sigma]\rho \implies (v_1, v_2) \in V[\sigma]\rho$

ii. $\forall v_1, v_2.(v_1, v_2) \in E[\tau]\rho \implies (v_1, v_2) \in V[\tau]\rho$

and prove

1. $\forall v_1, v_2.(v_1, v_2) \in E[\sigma \times \tau]\rho \implies (v_1, v_2) \in V[\sigma \times \tau]\rho$
2. $\forall v_1, v_2.(v_1, v_2) \in E[\sigma + \tau]\rho \implies (v_1, v_2) \in V[\sigma + \tau]\rho.$