Parametricity and Modular Reasoning
Homework #2

Instructor: Derek Dreyer

Assigned:	Tuesday, 6 November 2012
Problem 2:	Thursday, 8 November 2012
Due:	Tuesday, 13 November 2012

1 Canonical Forms for Sum Types

Prove that if $\vdash e : \tau_1 + \tau_2$, then $(e, e\,[\tau_1 + \tau_2]\,\mathrm{inj}_1\,\mathrm{inj}_2) \in E[\tau_1 + \tau_2]$.

2 Foldr-build Theorem

The foldr-build theorem underlies short-cut fusion. To state the theorem for System F extended with a base type $\mathrm{list}(\tau)$, we need Church-encoded lists:

$$\begin{aligned} \mathrm{Chlist}(\tau) &\stackrel{\mathrm{def}}{=} \forall \alpha. \alpha \to (\tau \to \alpha \to \alpha) \to \alpha \\ \mathrm{Chnil} &\stackrel{\mathrm{def}}{=} \Lambda \alpha. \lambda n. \lambda c. n \\ \mathrm{Chcons} &\stackrel{\mathrm{def}}{=} \lambda x. \lambda L. \Lambda \alpha. \lambda n. \lambda c. c\, x\, L. \end{aligned}$$

Prove that if $\vdash f : \mathrm{Chlist}(\tau)$ and $\vdash n : \sigma$ and $\vdash c : \tau \to \sigma \to \sigma$, then $\mathrm{fold}(n, c, \mathrm{build}\,\tau\,f) \equiv f\,\sigma\,n\,c : \sigma$ where

$$\begin{aligned} \mathrm{build}\,\tau\,f &\stackrel{\mathrm{def}}{=} f\,\mathrm{list}(\tau)\,\mathrm{nil}\,\mathrm{cons} \\ \mathrm{cons} &\stackrel{\mathrm{def}}{=} \lambda x. \lambda y. \mathrm{cons}(x, y). \end{aligned}$$

(We started a proof in class. See the November 8 scribe notes.)